**MATHEMATICAL LOGIC, ALGEBRA, AND NUMBER THEORY**

**1 GENERAL CHARACTERISTICS OF THE DISCIPLINE Mathematical Logic, Algebra, and Number Theory**

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| --- | --- |
| 1. Prerequisites | General algebra |
| 2. Corequisites | - |
| 3. Postrequisites | - |
| 4. Work input of the discipline module, credits | 3 |

**1.1. Objectives of the discipline**

The objectives of the discipline are

* To generate the final ideas about the role of general algebra, mathematical logic and number theory in other areas of mathematics;
* To generate the final ideas about the role of general algebra, mathematical logic and number theory in the development of mathematical models in other sciences.

The discipline is aimed at letting students study the competencies as follows:

The discipline is aimed at letting students study the competencies as follows:

* the ability to critically analyse and evaluate current scientific achievements, generate new ideas in solving research and practical problems, also in inter-disciplinary areas (Universal Competence-1);
* the ability to design and implement the integrated research including inter-disciplinary one based on a holistic and scientific view of the world using knowledge in the field of history and philosophy of science (Universal Competence-2);
* the readiness to be involved in the Russian and international research teams in addressing research and academic issues (Universal Competence-3);
* the ability to plan and solve own professional and personal development problems (Universal Competence-5);
* the ability to independently carry out research activities in the relevant professional field by applying modern research methods as well as information and communication technologies (General Professional Competence-1);
* the ability to professionally communicate the research outputs and present them in the form of scientific publications, information and analytical materials and presentations (General Professional Competence-2);
* understanding the role and place of the general algebra and mathematical logic in mathematics, their connection with other branches of mathematics and other fields of science (Professional Competence-1);
* the ability to study sets with algebraic operations and relations given on them (Professional Competence-2);
* the ability to study the syntactic and semantic properties of the formalised mathematical theories and the structural properties of their semantic models (Professional Competence-3);
* the ability to study algorithmic processes with given properties (Professional Competence-4)

**1.2. Discipline outcome requirements**

As a result of mastering the discipline, a student should:

Know:

- the structural theory of the basic types of universal algebras (groups, semigroups, domains, lattices) and the present state of universal algebra (the theory of hand congruences);

- the classical sections of the mathematical logic (propositional logic, predicate logic and computability theory);

- the basis of the contemporary number theory (arithmetic of integers, continued fractions, the properties of algebraic and transcendental numbers, the properties and applications of the basic number-theoretical functions).

Be able:

* to explore the main types of algebraic structures;
* to apply the general algebra methods and mathematical logic to the analysis of structures arising in other areas of mathematics and applications;
* to develop algorithms based on algebraic and logical models and analyse their complexity.

Master:

* the modern general algebra methods;
* the modern mathematical logic methods;
* the basic methods of number theory,

**1.3. Brief description of the discipline**

The discipline provides a summary. In studying it, the accumulated knowledge and skills associated with the place and role of the general algebra, mathematical logic and number theory in contemporary mathematics and other fields of knowledge are summarised.

***The proportion of sessions conducted in interactive forms:***

The proportion of sessions conducted in an interactive form is 100% of the volume of the in-class study workload by discipline.

1.4. Work intensity in mastering the discipline
In-class learning (intramural form of study)

|  |  |  |
| --- | --- | --- |
| Types of the educational work, forms of control | Total, hours | Number of the academic semester |
| 6 |
| In-class learning, hours | 4 | 4 |
| Lectures, hours | 4 | 4 |
| Practical exercises, hours |  |  |
| Laboratory-based work, hours |  |  |
| Self-guided work of students, hours | 104 | 104 |
| Type of interim assessment (credit/test, exam) | T | T |
| Total work intensity according to the curriculum, hours | 108 | 108 |
| Total work intensity according to the curriculum, credits | 3 | 3 |

**1. CONTENTS OF THE DISCIPLINE**

|  |  |  |
| --- | --- | --- |
| **Section code** | **Discipline section** | **Contents\*** |
| **Р1** | **Mathematical logic** | The logic of propositions, the logic of predicates, the theory of computability |
| **Р2** | **General algebra** | Groups, semigroups, domains, lattices, theory of hand congruences |
| **РЗ** | **Numbers theory** | Arithmetic of integers, continued fractions, properties of algebraic and transcendental numbers, properties and applications of basic number-theoretic functions |

**3. DISTRIBUTION OF THE LABOUR INTENSITY IN MASTERING THE DISCIPLINE BY SECTIONS AND CONTROL ACTIVITIES**

(Full-time study)

Learning semester 6 Scope of discipline (credits) 3

|  |  |  |
| --- | --- | --- |
| Discipline section | In-class load(hours) | Type, quantity and volumes of activities |
| Code of section, topic | Name of section, topic | Total of section, topic (hours) | Total | Lectures | Practical exercises | Laboratory-based work | Preparation for in-class learning (hours) | Total (hours) | Performing independent extracurricular activities (quantity) | Total (hours) | Preparation for the control qualification activities (quantity) |
| Total | Lectures | Pract. seminar classes | Laboratory-based work | Research seminars, conference seminars and colloquiums | Homework\* | Graphical work\* | Research paper, essay, creative work\* | Individual or group project\* | Translation of foreign literature\* j | Calculation wok, programme development\* | Calculation and graphical work\* | Term paper/ multi-disciplinary term work\* | Term paper/ multi-disciplinary term project\* | Review work (test)\* | Colloquium \* | Credit/test\* (given there is an exam) | Credit/test\* (graded given there is no exam) | Exam\* |
| P1 | Mathematical logic | 17 | 1 | 1 |  |  |  | 4 |  |  |  | 12 |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  | 1 |
| P2 | General algebra | 38 | 2 | 2 |  |  |  | 8 |  |  |  | 28 |  |  | 2 |  | 1 |  |  |  |  |  |  |  |
| P3 | Numbers theory | 17 | 1 | 1 |  |  |  | 4 |  |  |  | 12 |  |  |  |  | 1 |  |  |  |  |  |  |  |
|  | **Discipline, total (hours)** | 108 | 72 - |  |  | 36 |

**4 ORGANISATION OF PRACTICAL CLASSES AND SELF-GUIDED WORK OF STUDENTS ON THE DISCIPLINE**

**4.1. Laboratory**

not available

**4.2. Practical**

Not applicable

**4.3. Self-guided students’ work**

***4.3.1. An indicative list of the topics of research papers***

1. Contemporary problems of the theory of groups
2. Contemporary problems of the theory of semi-groups
3. Contemporary problems of the theory of domains
4. Contemporary problems of the lattice theory
5. Contemporary problems of the universal algebra
6. Non-negative matrices and their applications
7. Contemporary problems of the set theory
8. Contemporary problems of the mathematical logic
9. Non-classical logic
10. Contemporary problems of the theory of algorithms
11. Computational complexity of classical algorithms
12. Algorithms on graphs
13. Counts, matroids and their applications
14. Diophantine numbers and transcendental equations

***4.3.2. An indicative list of homework topics***

Not applicable

**4.3.3. An indicative list of the topics of tests**

Not applicable

***4.3.4. An indicative list of the topics of calculation works***

Not applicable

***4.3.5. An indicative list of the topics of calculation and graphic works***

Not applicable

***4.3.6. Sample topics of the colloquiums***

Not applicable

***4.3.7. Sample topics of term projects***

Not applicable

**4.4. An indicative list of test questions for preparation for the interim discipline assessment**

1. Sylow sub-groups, *p*-groups. The Sylow theorem. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, PK-2, Professional Competence-3, Professional Competence-4)
2. Finitely generated Abelian groups. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, PC-1, PC-2, PC-3, PC-4)
3. Nilpotent groups. The central chains of their properties. Common properties of nilpotent groups. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
4. Solvable groups. Finite solvable groups. Solvable groups of matrices. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
5. Characterisations of inverse semi-groups. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
6. Periodic semi-groups. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
7. Finitely based semi-groups. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
8. Eilenberg’s theorem on rational languages. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
9. Wedderburn Structure Theorem. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
10. Separable algebras over fields. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
11. Kaplansky-Amitsur theorem (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
12. Group P1-algebras. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
13. Algebraic lattices. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
14. Congruences in lattices, the lattice of a congruent lattice. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, General Professional Competence-3, General Professional Competence-4)
15. Semi-modular lattices, the Jordan-Hölder theorem. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
16. Lattices of partitions, their properties. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1. General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
17. Birkhoff’s theorem. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
18. Mal'tsev’s theorem on the structure of congruence-permutational varieties. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
19. Jonsson’s theorem, Day’s theorem. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
20. The structure of minimal algebras. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
21. Theorems on the completeness of the calculus of propositions, on adequacy, on feasibility, on compactness. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
22. The theorem on the feasibility of a consistent set of sentences, the completeness theorem, the adequacy theorem, and the compactness theorem. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
23. Expressiveness of polynomial queries, Chandra-Harell’s theorem on the expressibility of polynomial queries in a signature with linear order. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
24. The method of resolutions for the logic of predicates, its application. Completeness of the method of resolutions, the Erbran universe, semantic trees, the Erbran theorem. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4);
25. Decomposition of numbers into continued fractions. Approximation of numbers by continued fractions. Periodicity of continued fractions. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
26. Riemann zeta-function, its properties and applications. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)
27. Euler's theorem and Fermat's theorem. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1 Professional Competence-2, Professional Competence-3 Professional Competence-4)
28. Liouville numbers. Algebraic numbers, their properties. (Universal Competence-1, Universal Competence-2, Universal Competence-3, Universal Competence-5, General Professional Competence-1, General Professional Competence-2, Professional Competence-1, Professional Competence-2, Professional Competence-3, Professional Competence-4)

**Databases, information and reference systems and search systems**

1. The official Internet portal of legal information. – Available at http://pravo.gov.ru/, free. – Title from the screen.
2. Portal of information and educational resources of UrFU. - Available at http://study.urfu.ni/info/; free. – Title from the screen.
3. Electronic base of the regulatory documents of GOSTEXPERT, - Available at http://gostexpert.ru/, free, - Title from the screen.
4. Search engines: [www.yandex.ru](http://www.yandex.ru), google.ru, www.rambler.ru,