**ADDITIONAL CHAPTERS OF NUMERICAL METHODS**

**1 GENERAL CHARACTERISTICS OF the course *ADDITIONAL CHAPTERS OF NUMERICAL METHODS***

|  |  |
| --- | --- |
| 1. Pre-requisites | History of science  Research methodology |
| 2. Co-requisites | - |
| 3. Post-requisites | Final state attestation |
| 4. Complexity of the module course, credits | 3 |

1.1. Objectives of the course

The course objectives are:

- to let PhD students learn the modern section of numerical methods, the theory of solving ill-posed problems;

- the study of the key sections of mathematics application as the concept of ill-posedness, ways of regulating numerical differentiation problems, solving ill-conditioned systems of linear equations, extremal problems, integral equations of the first kind, linear and convex programming.

The course is aimed at enabling students to generate the following competencies:

- the ability to critically analyse and evaluate current scientific achievements, generate new ideas in solving research and practical problems including interdisciplinary areas (UK-1);

- readiness to participate in the work of Russian and international research teams in solving scientific and scientific-educational problems (Cross-Functional Competence-3);

- readiness to use modern methods and technologies of scientific communication in the state and foreign languages (Cross-Functional Competence-4);

- the ability to plan and solve problems of their own professional and personal development (Cross-Functional Competence-5);

- the ability to independently carry out research activities in the relevant professional field using up-to-date research methods as well as information and communication technologies (General Professional Competence-1);

- the readiness for teaching activities in the basic educational programmes for higher education (General Professional Competence-2).

- the capacity for intensive research and exploratory activities (Professional Competence-3);

- an independent analysis of physical aspects in the classical statement of mathematical problems (Professional Competence-4);

- the ability to navigate in modern algorithms of computer mathematics, improve, deepen and develop the mathematical theory underlying them (Professional Competence-7);

- the own vision of the applied aspect in strict mathematical formulations (Professional Competence-8);

- the ability to use, develop and implement mathematically complex algorithms in modern software systems (Professional Competence-9);

- the ability to represent and adapt mathematical knowledge in various ways taking into account the level of the audience (Professional Competence-11);

- the ability to manage and guide the research work of a team (Professional Competence-12);

- the ability to apply basic models and algorithms of computational mathematics to solving applied problems (Professional Competence-14);

- the ability to develop, analyse and justify the adequacy of mathematical models (Professional Competence-15);

- the ability to perform comparative analysis and implement a reasonable choice of algorithmic and software and hardware (Professional Competence-16);

- the ability to model and design data and structure knowledge as well as the applied and information processes (Professional Competence-17).

- the ability to use the basic natural science laws, apply the mathematical meta-language in professional activities, identify the essence of problems arising in the course of professional activities (Professional Competence-18);

- the ability to understand the essence and importance of information in the development of modern society, apply the achievements of computer science and computer technology, process large information amounts, conduct a targeted search in various sources of information regarding the profile activities including global computer systems (Professional Competence-19);

1.2. Criteria for the course outcomes

As a result of mastering the course, a student should:

**Know:**

* the current state and trends in the development of the theory of ill-posed problems;
* the possibility of using the theory of ill-posed problems for mathematical modelling and further use of methods of the theory of ill-posed problems in their professional activities;
* the key scientific achievements in the field of the theory of ill-posed problems, both fundamental and applied.

**Be able to:**

* to operate with the modern apparatus of the theory of ill-posed problems;
* Conduct scientific research using both classical and modern sections of the theory of ill-posed problems.

**Master:**

* the key theoretical provisions of the theory of ill-posed problems that are included in the programmes for qualifying exams for admission to post-graduate study;
* the methods of analysing the theory of ill-posed problems which use the modern apparatus of fundamental courses, especially a functional analysis;
* the qualitative research methods for the theory of ill-posed problems including the analysis of complex dynamic objects;
* methods of approximate study of the theory of ill-posed problems;
* computer technologies for the implementation of numerical algorithms for investigating the behaviour of systems of the complex nature.

1.2. Brief description of the course

The course addresses such important sections of mathematics as the notion of ill-posedness, methods of regulating numerical differentiation problems, solving ill-conditioned systems of linear equations, extremal problems, integral equations of the first kind, linear and convex programming.

The aim of the course is to introduce to students the foundation of the theory and modern methods of solving non-linear ill-posed problems.

The main goal of the course is to introduce students to the problem of a very important section of modern computational mathematics and functional analysis so that they can study the main classes of non-linear ill-posed problems and master approaches to constructing effective algorithms for solving them.

The proportion of classes conducted in interactive forms;

The proportion of classes conducted in an interactive form is 100% of the classroom workload by course.

1.3. The distribution of study hours for the course

Full-time education

|  |  |  |
| --- | --- | --- |
| Types of educational work, forms of control | Total, hours | Semester number |
| 3 |
| Face-to-face learning, hours | 4 | 4 |
| Lectures, hours | 4 | 4 |
| Practical exercises, hours |  |  |
| Laboratory-based work, hours |  |  |
| Self-study, hours | 104 | 104 |
| Interim assessment (test, exam) | 3 | 3 |
| Total workload according to the curriculum,  hours | 108 | 108 |
| Total workload according to the curriculum,  credits | 3 | 3 |

2. Content of the course

|  |  |  |
| --- | --- | --- |
| **Code of sections and topics** | Section, topic of the course**\*** | **Content** |
| Р1 | Examples of unstable problems. | Ill-conditioned systems of linear algebraic equations (SLAE), numerical differentiation and problems of restoring functions, extremal problems for convex functional, linear integral equations of convolution type, Volterra-Fredholm of the first kind, non-classical problems of mathematical physics, tomography problems, inverse problems for differential equations. |
|  |  |
| **Р2** | Ill-posed problem concept. | The concept of an ill-posed problem. Two basic statements, the solution to an equation and the evaluation of an unbounded operator. Correctness according to Hadamard, Tikhonov and Fiker. Sets of well-posedness. The Hausdorff lemma and its generalisations in the linear case. |
| **РЗ** | The concept of ill- and well-posed problems (based on the example of solving the SLAE problem) | The concept of ill- and well-posed problems (based on the example of solving the SLAE problem)  The SLAE conditionality measure. A calculation error analysis. A conditionality number as well as calculation (evaluation) properties and methods. SLAE conditionality and the stability of inverse matrices. The Hilbert matrix and other examples. Generalised solutions (by the example of the SLAE problem solution) Generalisation of the concept of solution. Pseudo-solution. Analysis of the method of least squares. A regular method for finding a pseudo-solution. The Tikhonov regularisation method. Regularisation methods (based on iterative algorithms and singular decomposition) Dual variational methods and iterative processes for solving ill-conditioned by SLAE. The singular decomposition method and its regularisation. Iterative refinement of the solution and the conditionality number. The tactics of solving ill-conditioned by SLAE and an analysis of software tools.  The concept of an optimal regularisation method. |
|  |  | The problem of an optimal regulariser when computing the values of an unbounded operator. Accuracy assessment. |
| **Р4** | The numerical differentiation problem | Regular algorithms for the numerical differentiation problem, mesh schemes in C(-oo, oo) method of mean functions. Interpolating and smoothing splines. Abstract splines. A comparative analysis of the effectiveness of various numerical differentiation methods. |
| **PS** | Extremum problems | Statement of the extremum problem of a functional. Correctness according to Hadamard and Tikhonov and their interrelation. Sufficient conditions for correctness according to Tikhonov. Regularisation with exact and approximate data. |
| **Р6** | Regularisation of convex problems | The method of penalty functions and the regularization of the convex programming problem in the general case. Discrete convergence of algorithms. Apparatus of discrete approximation and discrete convergence. Sufficient conditions for the convergence of discrete approximations in optimisation problems. Finite-dimensional approximation of RA. Appendix to the variational calculus, the rationale for the Ritz and Euler methods and finite-difference method. |
| **Р7** | Equations of the first kind. | Conditions of correctness of the operator equations of the first kind.  Equations generated by the Fredholm and Volterra integral operators and analysis of their correctness. Equations with apriori information.  Regularisation of equations of the first kind. Tikhonovskaya Regularisation of incorrect tasks. |
|  |  | Methods for regularisation of equations of the convolution type. Iterative regularisation. Iteration stopping rules, a-processes and their regularised analogues. Non-linear iterative processes for solving problems with a priori information. Finite-dimensional approximation of regularising algorithms. Finite-dimensional approximation. A convergence criterion. Applications of the general scheme: quadrature method, collocation method, projection methods. Theorems of convergence. Methods of self-regulation for Volterra equations.  Practical recommendations. A comparative analysis of the effectiveness of regular numerical methods for solving integral equations of the first kind. |
| **Р8** | Examples of non-linear unstable problems | Examples of non-linear unstable problems Operator and integral equations of the first kind (the problems of gravimetry and optics), optimisation problems for the convex functional, variational inequalities. Inverse problems for differential equations. Interrelation of different settings. |
| **Р9** | Fixed point principle | The principle of a fixed point. The classical principles of a fixed point by Banach, Brauer, Schauder and Kakutani. The principle of a fixed point by Browder for non-expansive mappings. Application of the fixed point principle for correct problems. Basic classes of non-linear mappings and their interrelation. Convergence of the method of successive approximations (msa) in the correct case. Weak convergence of iterations for pseudocompressive mappings.  Suggect correction |
| **Р10** | Method of correcting factors | The method of correcting factors. Regularisation of the method of successive approximations using corrective factors. Browder-Halperin approach and its generalisation. |
|  |  | Application to the mathematical programming problems. Regularisation of mathematical programming methods (prox-method, method of gradient projections, Fejér processes for convex inequality systems).  Application to linear equations. Iterative processes for solving linear ill-posed problems with a priori convex constraints. |
| **Pll** | Iterative regularisation principle | The iterative regularisation principle. The iteratively regularised method of successive approximations and its applications. The Polyak-Bakushinsky approach.  Application of the iterative regularisation principle. The iteratively regularised Newton-Kantorovich method with a monotonic operator. The regularised versions of the Guss-Newton method. The regularised properties of Mann processes for monotone operators.  Methods for the regularisation of spectral problems. Determination of the L-basis of the kernel of a linear operator. Stable methods of finding it. |
| **P12** | Some coefficient inverse problems | Some coefficient inverse problems. Inverse problems for ordinary differential equations, inverse coefficient problems for partial differential equations: the problem of the solution uniqueness. |
| **P13** | Methods for solving nonlinear equations with a priori information | Methods for solving non-linear equations with a priori information. Modification of the iterative regularisation methods by applying Feyer mappings. Applications.  The sub-gradient and sub-differential concepts. The problem of non-smooth minimisations of a convex functional. Iterative processes of the sub-gradient type |
| **Р14** | Explicit iterative processes for solving non-linear equations of the first kind | Explicit iterative processes for solving non-linear equations of the first kind. Convergence theorems for Landweber methods, steepest descent methods, and minimal error. Analysis of the experience of solving nonlinear integral equations of Fredholm and Volterra arising in the inverse problems of natural science. |

**3. DISTRIBUTION OF THE COURSE HOURS BY SECTIONS AND TEST ACTIVITIES**

(intramural form of study)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Course section | | | Face-to-face  (hours) | | | | Type, quantity and volumes of activities | | | | | | | | | | | | | | | | | | | | | | | | |
| Code of section, topic | Name of section, topic | Total of section, topic (hours) | Total | Lectures | Practical exercises | Laboratory-based work | Preparation for in-class learning (hours) | | | | | Total (hours) | Performing independent extracurricular activities (quantity) | | | | | | | | | | | Total (hours) | Preparation for the control and qualification activities (quantity) | | | | | | |
| Total | Lectures | Pract. seminar classes | Laboratory-based work | Research seminars, conference seminars and colloquiums | Homework\* | Graphical work\* | Research paper, essay, creative work\* | Individual or group project\* | | Translation of foreign literature\* | | Calculation wok, programme development\* | Calculation and graphical work\* | Term paper/ multi-disciplinary term work\* | Term paper/ multi-disciplinary term project\* | Review work (test)\* | Colloquium \* | Credit/test\* (given there is an exam) | | Credit/test\* (graded given there is no exam) | | Exam\* |
|  | Examples of unstable problems | 10 | 2 | 2 |  |  | 2 | 2 |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Concept of ill-posed problem | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Concept of ill-conditioned problems and well-defined problems (for example, the problem of solving SLAE) | 12 | 0 |  |  |  | 0 |  |  |  |  | 12 | 2 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Numerical differentiation problem | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Problems for the extremum | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Regularisation of convex problems | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Equations of the first kind | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Examples of non-linear unstable problems | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Fixed point principle | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Method of correcting factors | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Principle of iterative regularisation | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |
|  | Some coefficient inverse problems | 6 | 0 |  |  |  | 0 |  |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |
|  | Methods for solving non-linear equations with apriori information | 12 | 0 |  |  |  | 0 |  |  |  |  | 12 | 2 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Explicit iterative processes for solving non-linear equations of the first kind | 10 | 2 | 2 |  |  | 2 | 2 |  |  |  | 6 | 1 |  |  |  | |  | |  |  |  |  | 0 |  |  |  |  | |  | |
|  | Course, total (hours) | 104 | 4 | 4 | 0 | 0 | 4 | 4 | 0 | 0 | 0 | 96 | 96 | 0 | 0 | | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | | 4 | |

**4. ORGANISATION OF PRACTICAL LESSONS, SELF-STUDY AND ASSESSMENT**

4.1 Laboratory workshop

Not provided.

4.2 Practical classes Not provided

4.3 Independent work of students and MID-TERM assessment

4.3.1. An indicative list of topics for research papers

Not provided

3.3.2. An indicative list of homework topics

As homework, the postgraduate’s report can be prepared with some previous research on the following topics:

- Ill-conditioned systems of linear algebraic equations (SLAE).

- Numerical differentiation and the problem of restoring functions.

- Extreme problems for a convex functional.

- Linear integral equations of the convolution type, Volterra and Fredholm of the first kind.

- The concept of an ill-posed problem.

- Correctness according to Hadamard, Tikhonov and Fiker.

- Sets of correctness.

- The Hausdorff lemma and its generalisations in the linear case.

- The SLAE conditionality measure.

- Analysis of calculation error.

- Conditionality number, calculation (estimation) properties and methods.

- Conditionality of SLAE and stability of inverse matrices. The Hilbert matrix and other examples.

- Generalisation of the concept of solution.

- Pseudo-solution.

- Analysis of the method of least squares.

- Regular method for finding a pseudo-solution.

- The Tikhonov regularisation method.

- Dual variational methods and iterative processes for solving ill-conditioned SLAE.

- The singular decomposition method and its regularisation.

- Iterative refinement of the solution and the number of conditionality.

- Tactics of solving ill-conditioned SLAE and software tool analysis.

- The problem of the optimal regulariser in calculating the values of an unbounded operator. Estimation of error.

- The numerical differentiation problem

- Regular algorithms for the numerical differentiation problem, mesh schemes in ,C (-oo, oo) method of mean functions.

- Interpolating and smoothing splines.

- Abstract splines.

Comparative analysis of the effectiveness of various numerical differentiation methods.

Statement of the functional extremum problem.

Correctness according to Hadamard and Tikhonov and their interrelation.

Sufficient conditions for correctness according to Tikhonov.

Regularisation with the exact and approximate data.

The penalty function method and the regularisation of the convex programming problem in the general case.

Discrete convergence of algorithms.

Apparatus of discrete approximation and discrete convergence.

Sufficient conditions for the convergence of discrete approximations in optimisation problems.

Finite-dimensional approximation of PA.

Appendix to the variational calculus, the rationale for the Ritz and Euler methods and that for the finite-difference method.

Conditions for the correctness of operator equations of the first kind.

Equations generated by Fredholm and Volterra integral operators and analysis of their correctness.

Equations with a priori information.

Regularisation of equations of the first kind.

Tikhonov regularisation of ill-posed problems.

Convolutional equation regularisation methods.

Iterative regularisation. Iteration stopping rules, a-processes and their regularised analogues.

Non-linear iterative processes for solving problems with a priori information.

Finite-dimensional approximation. A convergence criterion.

Applications of the general scheme, such as quadrature method, collocation method, projection methods.

Convergence theorems. Self-regulation methods for Volterra equations. Comparative analysis of the effectiveness of regular numerical methods for solving integral equations of the first kind.

Operator and integral equations of the first kind (gravimetry and optics problems), optimisation problems for a convex functional, variational inequalities.

Inverse problems for differential equations. Interrelation of different productions.

Classical principles of the fixed point of Banach, Brauer, Schauder and Kakutani. The principle of the fixed Browder point for non-expansive mappings. Application of the fixed point principle for correct problems. Basic classes of non-linear mappings and their interrelation.

Convergence of the method of successive approximations in the correct case. Weak convergence of iterations for pseudo-compressive mappings.

Regularization of successive approximations using corrective multipliers. The Browder-Halperin approach and its generalisation.

Regularisation of mathematical programming methods (pro-method, method of gradient projections, Fejér processes for systems of convex inequalities). Iterative processes for solving linear ill-posed problems with a priori convex constraints.

Iteratively regularised successive approximations and its applications. The Polyak-Bakushinsky approach.

Iteratively regularised Newton-Kantorovich method with monotone operator.

Regularised versions of the Gauss-Newton method. Regularising properties of Mann processes for monotone operators.

Methods for the regularisation of spectral problems. Determination of the L-basis of the kernel of a linear operator. Stable methods for finding it.

Inverse problems for ordinary differential equations, inverse coefficient problems for partial differential equations, the problem of uniqueness of a solution.

Methods for solving non-linear equations with a priori information. Modi

4.3.3. An indicative list of control topics

Not provided

4.3.4. An indicative list of the topics of calculation works

Not provided

4.3.5. An indicative list of topics for calculation and graphic work

Not provided

4.3.6. Sample topics of the colloquia

Not provided

4.3.2. Sample topics of the course project (work)

Not provided

4.4 An indicative list of control questions to prepare for the course qualification

- Ill-conditioned systems of linear algebraic equations (SLAE).

- Numerical differentiation and the function restoring problem.

Extremal problems for a convex functional.

- Linear integral equations of the convolution type, Volterra and Fredholm of the first kind.

- The concept of an ill-posed problem.

- Correctness according to Hadamard, Tikhonov and Fiker.

Correctness sets.

- Hausdorff’s lemma and its generalisations in the linear case.

- The SLAE conditionality measure.

Analysis of calculation error.

- Condition number, calculation (estimation) properties and methods.

Conditionality of SLAE and stability of inverse matrices. The Hilbert matrix and other examples.

Generalisation of the concept of solution.

Pseudo-solution.

Analysis of the method of least squares.

Regular method for finding a pseudo-solution.

The Tikhonov regularisation method.

Dual variational methods and iterative processes for solving ill-conditioned SLAE.

The singular decomposition method of and its regularisation.

Iterative improvement of a solution and the conditioning number.

The tactics of solving ill-conditioned SLAE and the analysis of software tools. The problem of an optimal regulariser when computing the values of an unbounded operator. Estimation of error.

The numerical differentiation problem

Regular algorithms for the numerical differentiation problem, mesh schemes in ,C (-oos co) method of mean functions.

Interpolating and smoothing splines.

Abstract splines.

Comparative analysis of the effectiveness of various numerical differentiation methods.

Statement of the problem on the extremum of a functional.

Correctness according to Hadamard and Tikhonov and their interrelation.

Sufficient conditions for correctness according to Tikhonov.

Regularisation with exact and approximate data.

The penalty function method and the regularisation of the convex programming problem in the general case.

Discrete convergence of algorithms.

Discrete approximation and discrete convergence apparatus.

Sufficient conditions for the convergence of discrete approximations in optimisation problems.

Finite-dimensional approximation of PA.

Application to the variational calculus, the rationale for the Ritz and Euler methods, the finite-difference method.

Conditions for the correctness of operator equations of the first kind.

Equations generated by Fredholm and Volterra integral operators and analysis of their correctness.

Equations with apriori information.

Regularisation of equations of the first kind.

Tikhonov regularisation of ill-posed problems.

Methods for the regularisation of convolutional equations.

Iterative regularisation. Rules for stopping iterations. A-processes and their regularised analogues.

Non-linear iterative processes for solving problems with apriori information.

Finite-dimensional approximation. A criterion for convergence.

Applications of the general scheme, a quadrature method, collocation method, projection methods.

Theorems of convergence. Self-regulation methods of for Volterra equations. A comparative analysis of the effectiveness of regular numerical methods for solving integral equations of the first kind.

Operator and integral equations of the first kind (problems of gravimetry and optics), optimisation problems for a convex functional, variational inequalities.

Inverse problems for differential equations. Interrelation of different productions.

Classical principles of the fixed point of Banach, Brauer, Schauder and Kakutani. The principle of the fixed Browder point for non-expanding mappings. Application of the fixed point principle for correct problems. Basic classes of non-linear mappings and their interrelation.

Convergence of the method of successive approximations in the correct case. Weak convergence of iterations for pseudo-contracting mappings.

Regularisation of the method of successive approximations using corrective multipliers. The Browder-Halperin approach and its generalisation.

Regularisation of methods of mathematical programming (pro-method, method of gradient projections, Fejér processes for systems of convex inequalities). Iterative processes for solving linear ill-posed problems with a priori convex constraints.

The iteratively regularised method of successive approximations and its applications. The Polyak-Bakushinsky approach.

The iteratively regularised Newton-Kantorovich method with the monotonic operator.

The regularised versions of the Gauss-Newton method. Regularising properties of the Mann processes for monotone operators.

Methods for the regularisation of spectral problems. Determination of the L-basis of the kernel of a linear operator. Stable methods of finding it.

- Inverse problems for ordinary differential equations, inverse coefficient problems for partial differential equations, the problem of uniqueness of a solution.

- Methods for solving non-linear equations with apriori information. Modification of iterative regularisation methods through Feyer mappings.

- The concepts of sub-gradient and sub-differential. The problem of non-smooth minimisation of a convex functional.

- Iterative processes of the sub-gradient type.

- Explicit iterative processes for solving non-linear equations of the first kind

- Theorems of convergence for Landweber methods, methods of steepest descent and minimal error.

- Analysis of the experiment solutions of non-linear Fredholm and Volterra integral equations arising in the inverse problems of natural science.

43.3. An indicative list of topics

Not provided

43.4. An indicative list of the topics of calculation works

Not provided

4.3.5. An indicative list of topics for the calculation and graphical works

Not provided

4.3.6. Sample topics of the colloquia

Not provided

4.3.2. Sample topic of the course project (work)

Not provided

4L. An indicative list of control questions to prepare for the course qualification

- Ill-conditioned systems of linear algebraic equations (SLAE).

- Numerical differentiation and the problem of restoring functions.

Extremal problems for a convex functional.

- Linear integral equations of convolution type, Volterra and Fredholm of the first kind.

- The concept of an ill-posed problem.

- Correctness according to Hadamard, Tikhonov and Fiker.

Correctness sets.

- Hausdorff’s lemma and its generalisations in the linear case.

- The measure of conditionality of SLAE.

Analysis of calculation error.

- Condition number: properties and methods of calculation (estimation).

Conditionality of SLAE and the stability of inverse matrices. The Hilbert matrix and other examples.

Generalisation of the concept of solution.

Pseudo-solution.

Analysis of the method of least squares.

A regular method for finding a pseudo-solution.

The Tikhonov regularisation method.

Dual variational methods and iterative processes for solving ill-conditioned SLAE.

The method of singular decomposition and its regularisation.

Iterative improvement of the solution and that of the number of conditionality.

The tactics of solving ill-conditioned SLAE and a software tool analysis. The problem of an optimal regulariser in computing the values of an unbounded operator. Estimation of error.

The numerical differentiation problem

Regular algorithms for the numerical differentiation problem, mesh schemes in ,C (-oos co) method of mean functions.

Interpolating and smoothing splines.

Abstract splines.

A comparative analysis of the effectiveness of various numerical differentiation methods.

Statement of the problem on the extremum of a functional.

Correctness according to Hadamard and Tikhonov and their interrelation.

Sufficient correctness conditions according to Tikhonov.

Regularisation with the exact and approximate data.

The penalty function method and the regularisation of the convex programming problem in the general case.

Discrete convergence of algorithms.

Apparatus of discrete approximation and discrete convergence.

Sufficient conditions for the convergence of discrete approximations in optimisation problems.

Finite-dimensional approximation of PA.

Application to the calculus of variations, the rationale for the Ritz and Euler methods, the finite-difference method.

Conditions for the correctness of operator equations of the first kind.

Equations generated by Fredholm and Volterra integral operators and analysis of their correctness.

Equations with a priori information.

Regularisation of equations of the first kind.

Tikhonov regularisation of ill-posed problems.

Methods of regularisation of the convolutional equations.

Iterative regularisation. Rules for stopping iterations. A-processes and their regularised analogues.

Non-linear iterative processes for solving problems with a priori information.

Finite-dimensional approximation. A criterion for convergence.

Applications of the general scheme, a quadrature method, a collocation method and projection methods.

Theorems of convergence. Methods of self-regulation for Volterra equations. Comparative analysis of the effectiveness of regular numerical methods for solving integral equations of the first kind.

Operator and integral equations of the first kind (the problems of gravimetry and optics), optimisation problems for a convex functional, variational inequalities.

Inverse problems for differential equations. Interrelation of different productions.

Classical principles of the fixed point of Banach, Brauer, Schauder and Kakutani. The principle of the fixed Browder point for non-expanding mappings. Application of the fixed point principle for correct problems. Basic classes of non-linear mappings and their interrelation.

Convergence of the method of successive approximations in the correct case. Weak convergence of iterations for pseudo-contracting mappings.

Regularisation of successive approximations using corrective multipliers. The Browder-Halperin approach and its generalisation.

Regularisation of mathematical programming methods (prox method, method of gradient projections, Fejér processes for convex inequality systems). Iterative processes for solving linear ill-posed problems with a priori convex constraints.

The iteratively regularised method of successive approximations and its applications. The Polyak-Bakushinsky approach.

The iteratively regularised Newton-Kantorovich method with monotonic operator.

Regularised versions of the Gauss-Newton method. Regularising properties of the Mann processes for monotone operators.

Methods for the regularisation of spectral problems. Determination of the L-basis for the kernel of a linear operator. Stable methods for finding it.

Inverse problems for ordinary differential equations, inverse coefficient problems for partial differential equations, the problem of uniqueness of a solution.

Methods for solving non-linear equations with apriori information. Modification of the iterative regularisation methods through Feyer mappings. The concepts of sub-gradient and sub-differential. The problem of non-smooth minimisation of a convex functional,

- Iterative processes of the sub-gradient type.

Explicit iterative processes for solving non-linear equations of the first kind

- Theorems of convergence for Landweber methods, methods of steepest descent and minimal error.

- Analysis of the experience in solving nonlinear integral equations of Fredholm and Volterra arising in inverse problems of the natural science

As a qualification of the seminar, the post-graduate student’s report with the previous research work can be counted.

**5.1. Databases, information and reference systems and search systems**

1) The official Internet portal of legal information. - Access mode:

http://pravo.gov.ru/, free. - Title from the screen.

2) Portal of the UrFU information and educational resources. - Access mode:

http://study.urfu.ru/info/, free. - Title from the screen.

3) Electronic base of the regulatory documents of GOSTEXPERT. - Access mode:

http://gostexpert.ru/, free. - Title from the screen.

4) Search engines: www.yandex.ru, google.ru www.rambler.ru,