**DISCRETE MATHEMATICS AND MATHEMATICAL CYBERNETICS**

**1 GENERAL CHARACTERISTICS OF THE DISCIPLINE/COURSE**

***DISCRETE MATHEMATICS AND MATHEMATICAL CYBERNETICS***

|  |  |
| --- | --- |
| 1. Prerequisites | History of scienceMethodology of research |
| 2. Corequisites | - |
| 3. Postrequisites | - |
| 4. Work input of the course module, credits | 3 |

**1.1. Objectives of the course**

The objectives of the course are

* To develop the final ideas about the role of discrete mathematics and mathematical cybernetics in the development of mathematical models;
* To develop the final ideas about the role of discrete mathematics and mathematical cybernetics in the design of algorithms and programme packages.

The discipline is aimed at developing in students the following competencies:

* the ability to critically analyse and evaluate current scientific achievements, generate new ideas in solving research and practical problems, also in inter-disciplinary areas (Universal Competence-1);
* the ability to design and implement integrated research including inter-disciplinary one based on a holistic and scientific view of the world using knowledge in the field of history and philosophy of science (Universal Competence-2);
* the readiness to be involved in the Russian and international research teams in addressing research and academic issues (Universal Competence-3);
* the ability to plan and solve own professional and personal development problems (Universal Competence-5);
* the ability to independently carry out research activities in the relevant professional field by applying modern research methods as well as information and communication technologies (General Professional Competence-1);
* the ability to professionally communicate the research outputs and present them in the form of scientific publications, information and analytical materials and presentations (General Professional Competence-3);
* mastering mathematical modelling methods of in the analysis of global problems on the basis of deep knowledge of fundamental mathematical disciplines and computer sciences (Professional Competence-1);
* mastering the mathematical and algorithmic modelling methods in the analysis of natural science problems (Professional Competence-2);
* the ability to intensively carry out research and development (Professional Competence-3);
* the independent analysis of physical aspects in the classical statements of mathematical problems (Professional Competence-4);
* the ability to publicly present your own new research outcomes (Professional Competence-5);
* independent construction of an aggregate picture of the discipline (Professional Competence-6);
* the ability to find a way in modern algorithms of computer mathematics as well as improve, deepen and develop the mathematical theory underlying them (Professional Competence-7);
* own vision of the applied aspect in mathematical formulations (Professional Competence-8);
* the ability to use, develop and implement sophisticated mathematical algorithms in modern software systems (Professional Competence-9);
* definition of common forms, patterns and tools for groups of disciplines (Professional Competence-10);
* the ability to represent and adapt mathematical knowledge in various ways taking into account the audience level (Professional Competence-11);
* the ability to manage and guide the research work in teams (Professional Competence-12);
* the ability to formulate in the form of problems and assignments non-mathematical types of knowledge (including humanitarian) (PC-13);
* the ability to apply basic computational mathematics models and algorithms to solving applied problems (Professional Competence-14);
* the ability to develop, analyse and justify the adequacy of mathematical models (Professional Competence-15);
* the ability to perform a comparative analysis and reasonably choose algorithmic systems as well as software and hardware (Professional Competence-16);
* the ability to model and design data and knowledge structures, application and information processes (Professional Competence-17).
* the ability to use the basic natural science laws, apply the mathematical apparatus in professional activities, identify the essence of problems arising in the course of professional activities (Professional Competence-18);
* the ability to understand the essence and importance of information in the development of modern society, apply the achievements of computer science and computer technology, process large amounts of information, carry out the purposeful search in various sources of information on the activity profile including the global computer systems (Professional Competence-19);

**1.2. Discipline outcome requirements**

As a result of mastering the discipline, a student should:

Know:

- the classical theory of algorithms, fundamentals and the current state of the computational complexity theory;

- the classical sections of mathematical logic and its applications to the description of combinatorial problems;

- the key sections of graph theory including the theory of random graphs and the theory of expanders;

- the classical sections and the current state of the theory of formal languages, their acceptors and transducers (automata and machines), and their generators (grammars).

Be able:

* to build and explore the mathematical models of discrete processes using graphs, automata, languages, matroids and other discrete-mathematical objects;
* to design equivalent transformations between the logical, algebraic and combinatorial formalisms for mathematical problems;
* to develop algorithms for solving discrete problems and analyse their complexity.

Master:

* methods for constructing and evaluating algorithms;
* complexity theory methods;
* graph theory methods;
* methods of automata theory and formal language theory;
* methods of mathematical logic and computability theory.

**1.3. Brief description of the discipline**

The discipline Discrete Mathematics and Mathematical Cybernetics is intended for the preparation of graduate students for passing the PhD exam in the specialty 01.01.09 – Discrete Mathematics and Mathematical Cybernetics.

***The proportion of sessions conducted in interactive forms:***

The proportion of sessions conducted in an interactive form is 100% of the volume of the in-class study workload by discipline.

1.4. Work intensity in mastering the discipline
In-class learning

|  |  |  |
| --- | --- | --- |
| Types of tuition, forms of assessment | Total, hours | Number of the academic semester |
| 6 |
| Face-to-face learning, hours | 4 | 4 |
| Lectures, hours | 4 | 4 |
| Practical exercises, hours |  |  |
| Laboratory-based work, hours |  |  |
| Self-study, hours | 104 | 104 |
| Type of interim assessment (credit/test, exam) | E | E |
| Total work intensity according to the curriculum, hours | 108 | 108 |
| Total work intensity according to the curriculum, credits | 3 | 3 |

**2. CONTENTS OF THE COURSE**

|  |  |  |
| --- | --- | --- |
| Section code | Discipline section | Contents\* |
| Р1 | Mathematical logic | Boolean functions, first-order logic, computability theory, extensions of first-order logic, temporal logic |
| Р2 | Algorithms and complexity | Design and analysis of efficient algorithms, probabilistic and approximate algorithms, complexity theory and its applications |
| РЗ | Graph theory | Combinatorial graph theory, spectral graph theory, random graphs, expanders |
| Р4 | Automata and formal languages | Finite automata and regular languages, pushdown automata and context-free languages, extensions of context-free languages, languages and automata on trees |

**3. DISTRIBUTION OF THE HOURS BY SECTIONS AND ASSESSMENT**

(Full-time study)

Learning semester 6 Scope of discipline (credits) 3

|  |  |  |
| --- | --- | --- |
| Discipline section | In-class load(hours) | Type, quantity and volumes of activities |
| Code of section, topic | Name of section, topic | Total of section, topic (hours) | Total | Lectures | Practical exercises | Laboratory-based work | Preparation for face-to-face learning (hours) | Total (hours) | Performing independent extracurricular activities (quantity) | Total (hours) | Preparation for the control qualification activities (quantity) |
| Total | Lectures | Pract. seminar classes | Laboratory-based work | Research seminars, conference seminars and colloquiums | Homework\* | Graphical work\* | Research paper, essay, creative work\* | Individual or group project\* | Translation of foreign literature\* j | Calculation work, programme development\* | Calculation and graphical work\* | Term paper/ multi-disciplinary term work\* | Term paper/ multi-disciplinary term project\* | Review work (test)\* | Colloquium \* | Credit/test\* (given there is an exam) | Credit/test\* (graded given there is no exam) | Exam\* |
| P1 | Mathematical logic | 17 | 1 | 1 |  |  |  | 4 |  |  |  | 12 |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  | 1 |
| P2 | Algorithms and complexity | 19 | 1 | 1 |  |  |  | 4 |  |  |  | 14 |  |  | 1 |  |  |  |  |  |  |  |  |  |
| P3 | Graph theory | 17 | 1 | 1 |  |  |  | 4 |  |  |  | 12 |  |  |  |  | 1 |  |  |  |  |  |  |  |
| P4 | Theory of automata and formal languages | 19 | 1 | 1 |  |  |  | 4 |  |  |  | 14 |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |
|  | **Discipline, total (hours)** | 108 | 72 \_ - |  |  | 36 |

**4 ORGANISATION OF PRACTICAL CLASSES AND SELF-STUDY OF STUDENTS ON THE DISCIPLINE**

**4.1. Laboratory**

not available

**4.2. Practical**

not provided

**4.3. Independent work of students**

***4.3.1. An indicative list of the topics of research papers***

1. Contemporary problems of the theory of algorithms

2. Contemporary problems of complexity theory

3. Contemporary problems of the combinatorial theory of graphs

4. Graph spectra and their applications

5. Non-negative matrices and their applications

6. Random graphs and their applications

7. Expanders and their applications

8. Linear temporal logic

9. Synchronized automata

10. Repetition-free sequences

11. Transducers and rational relations

12. CSP problems and their complexity

13. SAT problems and basic approaches to SAT solving

14. Pattern matching algorithms

15. Burrows–Wheeler transform and its applications

***4.3.2. An indicative list of homework topics***

Not provided

**4.3.3. An indicative list of the topics of tests**

Not provided

***4.3.4. An indicative list of the topics of calculation works***

Not provided

4.3.5. An indicative list of the topics of calculation and graphic works

Not provided

***4.3.6. Sample topics of the colloquiums***

Not provided

***4.3.7. Sample topics of term projects***

Not provided

**4.4. An indicative list of test questions for preparation for the interim discipline assessment**

1. Eulerian cycle, Euler’s theorem for undirected and directed cases, complexity of algorithms searching for the Eulerian cycle.

2. Hamiltonian cycle (or Hamiltonian circuit), Ore and Hwatal’s theorems, the NP-completeness of the problem of finding a Hamiltonian cycle.

3. Planar graphs, minors and the planarity criterion.

4. Graph colouring, Brooks and Heawood’s theorems, NP-completeness of the 3-colouring problem.

5. Random graphs. Phase transition for the connectedness property.

6. Spectra of graphs. Coefficients of the Characteristic Polynomial.

7. Boolean functions. SAT problem and Cook’s theorem.

8. Variants of the SAT problem (k-SAT, MaxSAT, QSAT) and their complexity.

9. The CSP problem, its complexity and polynomial subclasses.

10. Boolean circuits and circuit complexity.

11. Resolution method in the first-order logic.

12. Gödel’s completeness theorem for the first-order logic.

13. Theorem of compactness of the first-order logic.

14. Gödel’s incompleteness theorem.

15. Monadic second order logic and regular languages.

16. Recursiveness and computability, equivalence of the Church and Turing theses.

17. NP and coNP complexity classes. Certificates, verification, asymmetry.

18. Log-space reductions. Completeness of decision problems.

19. Nivat’s theorem on finite transducers.

20. Trakhtman’s road colouring theorem.

21. PSPACE-complete problems for finite automata.

22. Context-free grammars and languages, the substitution theorem.

23. Context-free grammars and languages, the pumping lemma.

24. Pushdown automata and their computational power.

25. Hash functions, universal classes of hash functions.

26. Fast Fourier transform and its application to string algorithms.

27. Classification of NP-complete problems by approximability. The APX, PTAS, and FPTAS classes.

28. The construction of a fully polynomial time approximation scheme (FPTAS) for the knapsack problem.

29. Flows in graphs, the Ford–Fulkerson theorem and algorithm.

**5. METHODOLOGICAL AND INFORMATION SUPPORT TO THE DISCIPLINE**

Recommended literature

* J. Kleinberg, E. Tardos. Algorithm design. NY: Pearson, 2006.
* R. Diestel. Graph Theory. Springer, 2000.
* C. Papadimitriou. Computational Complexity. Addison-Wesley, 1995.
* V. R.L. Graham, M. Grotschel, L. Lovasz. Handbook of Combinatorics, Volume 1. Elsevier, 1995.
* J.L. Gross, J. Yellen. Handbook of Graph Theory. CRC Press, 2003.
* P.B. Andrews. An introduction to mathematical logic and type theory. Kluwer Acad. Publ., 2002.

**Databases, information and reference systems and search systems**

1. The official Internet portal of legal information. - Access mode: http://pravo.gov.ru/, free. - Title from the screen.

2. Portal of information and educational resources of the UrFU. - Available at http://study.urfu.ru/info/, free. – Title from the screen.

3. Electronic base of normative documents of GOSTEXPERT. - Available at http://gostexpert.ru/, free. - Title from the screen.

4. Search engines: www.yandex.ru, google.ru www.rambler.ru,

**c. Electronic learning resources**

1. All students have full access to the listed resources, also through the authorised access from the Internet:
2. Elsevier B.V. DB Reaxys Agreement No. 1-3839832505 dated February 20, 2013;
3. First Independent Rating Agency LLC IPA FIRAPRO Agreement No 43-12/370-2013 dated May 23, 2013;
4. EBSCO Industries, 1BDB Business Source Complete Agreement No 624 dated July 2, 2013;
5. EBSCO Industries, 1BDB EBSCO Discovery Service Agreement No 625 dated July 02, 2013;
6. Elsevier B.V. DB Freedom Collection Agreement No 1-4412061361 dated April 26, 2013;
7. NEICON non-commercial partnership, company DB Thomson Reuters, Web of science composed of DB Citation Index Expanded, DB Social Sciences Index, DB Art&Humanities Citation Index, Journal Citation Reports, Conference Proceedings Citation Index Agreement No 43-12/456-2013 dated July 12, 2013;

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